

## Viscosity and Density (Metric SI Units)

In the SI system of units the kilogram (kg) is the standard unit of mass, a cubic meter is the standard unit of volume and the second is the standard unit of time.

### **Density $\rho$**

The density of a fluid is obtained by dividing the mass of the fluid by the volume of the fluid. Density is normally expressed as kg per cubic meter.

$$\rho = \text{kg/m}^3$$

Water at a temperature of 20°C has a density of 998 kg/m<sup>3</sup>

Sometimes the term 'Relative Density' is used to describe the density of a fluid.

Relative density is the fluid density divide by 1000 kg/m<sup>3</sup>

Water at a temperature of 20°C has a Relative density of 0.998

### **Dynamic Viscosity $\mu$**

Viscosity describes a fluids resistance to flow.

Dynamic viscosity (sometimes referred to as Absolute viscosity) is obtained by dividing the Shear stress by the rate of shear strain.

The units of dynamic viscosity are: Force / area x time

The Pascal unit (Pa) is used to describe pressure or stress = force per area

This unit can be combined with time (sec) to define dynamic viscosity.

$$\mu = \text{Pa}\cdot\text{s}$$

$$1.00 \text{ Pa}\cdot\text{s} = 10 \text{ Poise} = 1000 \text{ Centipoise}$$

Centipoise (cP) is commonly used to describe dynamic viscosity because water at a temperature of 20°C has a viscosity of 1.002 Centipoise.

This value must be converted back to  $1.002 \times 10^{-3} \text{ Pa}\cdot\text{s}$  for use in calculations.

### **Kinematic Viscosity $\nu$**

Sometimes viscosity is measured by timing the flow of a known volume of fluid from a viscosity measuring cup. The timings can be used along with a formula to estimate the kinematic viscosity value of the fluid in Centistokes (cSt).

The motive force driving the fluid out of the cup is the head of fluid.

This fluid head is also part of the equation that makes up the volume of the fluid.

Rationalizing the equations the fluid head term is eliminated leaving the units of Kinematic viscosity as area / time

$$\nu = \text{m}^2/\text{s}$$

$$1.0 \text{ m}^2/\text{s} = 10000 \text{ Stokes} = 1000000 \text{ Centistokes}$$

Water at a temperature of 20°C has a viscosity of  $1.004 \times 10^{-6} \text{ m}^2/\text{s}$   
This evaluates to 1.004000 Centistokes.  
This value must be converted back to  $1.004 \times 10^{-6} \text{ m}^2/\text{s}$  for use in calculations.

The kinematic viscosity can also be determined by dividing the dynamic viscosity by the fluid density.

### ***Kinematic Viscosity and Dynamic Viscosity Relationship***

Kinematic Viscosity = Dynamic Viscosity / Density

$$v = \mu / \rho$$

Centistokes = Centipoise / Density

To understand the metric units involved in this relationship it will be necessary to use an example:

Dynamic viscosity  $\mu = \text{Pa}\cdot\text{s}$

Substitute for Pa =  $\text{N}/\text{m}^2$  and N =  $\text{kg}\cdot\text{m}/\text{s}^2$

Therefore  $\mu = \text{Pa}\cdot\text{s} = \text{kg}/(\text{m}\cdot\text{s})$

Density  $\rho = \text{kg}/\text{m}^3$

Kinematic Viscosity =  $v = \mu/\rho = (\text{kg}/(\text{m}\cdot\text{s}) \times 10^{-3}) / (\text{kg}/\text{m}^3) = \text{m}^2/\text{s} \times 10^{-6}$

### **Viscosity and Density (Imperial Units)**

In the Imperial system of units the pound (lb) is the standard unit of weight, a cubic foot is the standard unit of volume and the second is the standard unit of time.

The standard unit of mass is the slug.

This is the mass that will accelerate by 1 ft/s when a force of one pound (lbf) is applied to the mass. The acceleration due to gravity (g) is 32.174 ft per second per second.

To obtain the mass of a fluid the weight (lb) must be divided by 32.174.

#### ***Density $\rho$***

Density is normally expressed as mass (slugs) per cubic foot.

The weight of a fluid can be expressed as pounds per cubic foot.

$$\rho = \text{slugs}/\text{ft}^3$$

Water at a temperature of 70°F has a density of 1.936 slugs/ft<sup>3</sup> (62.286 lbs/ft<sup>3</sup>)

#### ***Dynamic Viscosity $\mu$***

The units of dynamic viscosity are: Force / area x time

$$\mu = \text{lb}\cdot\text{s}/\text{ft}^2$$

Water at a temperature of 70°F has a viscosity of  $2.04 \times 10^{-5} \text{ lb}\cdot\text{s}/\text{ft}^2$

$$1.0 \text{ lb}\cdot\text{s}/\text{ft}^2 = 47880.26 \text{ Centipoise}$$

### ***Kinematic Viscosity $\nu$***

The units of Kinematic viscosity are area / time

$$\nu = \text{ft}^2/\text{s}$$

$$1.00 \text{ ft}^2/\text{s} = 929.034116 \text{ Stokes} = 92903.4116 \text{ Centistokes}$$

Water at a temperature of 70°F has a viscosity of  $10.5900 \times 10^{-6} \text{ ft}^2/\text{s}$   
(0.98384713 Centistokes)

### ***Kinematic Viscosity and Dynamic Viscosity Relationship***

Kinematic Viscosity = Dynamic Viscosity / Density

$$\nu = \mu / \rho$$

The imperial units of kinematic viscosity are  $\text{ft}^2/\text{s}$

To understand the imperial units involved in this relationship it will be necessary to use an example:

Dynamic viscosity  $\mu = \text{lb}\cdot\text{s}/\text{ft}^2$

Density  $\rho = \text{slugs}/\text{ft}^3$

Substitute for slug =  $\text{lb}/32.174 \text{ ft}\cdot\text{s}^2$

Density  $\rho = (\text{lb}/32.174 \text{ ft}\cdot\text{s}^2)/\text{ft}^3 = (\text{lb}/32.174 \cdot \text{s}^2)/\text{ft}^4$

Note:  $\text{slugs}/\text{ft}^3$  can be expressed in terms of  $\text{lb}\cdot\text{s}^2/\text{ft}^4$

Kinematic Viscosity  $\nu = (\text{lb}\cdot\text{s}/\text{ft}^2)/(\text{slugs}/\text{ft}^3)$

Substitute  $\text{lb}\cdot\text{s}^2/\text{ft}^4$  for  $\text{slugs}/\text{ft}^3$

Kinematic Viscosity  $\nu = (\text{lb}\cdot\text{s}/\text{ft}^2) / (\text{lb}\cdot\text{s}^2/\text{ft}^4) = \text{ft}^2/\text{s}$